Competitive Math Sample Advanced Topics

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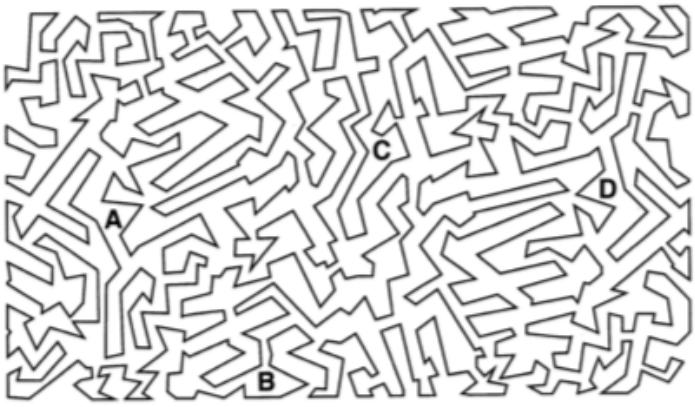
1 Fundamentals 2

1.1 Cryptarium

Problem 1. Solve the Cryptarium puzzle below.

$$\begin{array}{cccc} & A & A & A \\ & & B & B \\ + & & & A \\ \hline & C & A & B \end{array}$$

1.2 Jordan Curve Theorem

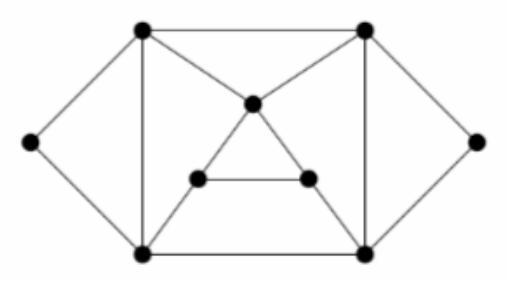


Problem 2. Label whether each of the points A, B, C, and D is ***inside*** or ***outside*** of the shape above.[1]

[1] Beast Academy Practice 2C

1.3 Eulerian Paths

Problem 3. In a diagram of dots and lines, a path that traces every line exactly once without picking up your pencil or tracing over the same line twice is called an Eulerian path. For the diagram below, is it possible to draw an Eulerian path? If it is possible, draw an Eulerian path. Otherwise, explain why it is not possible.

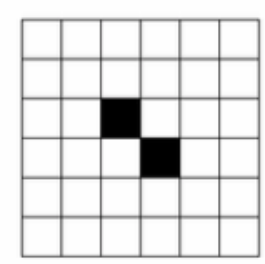


1.4 Problem Solving

Problem 4. Four people come to a river in the night. There is a narrow bridge connecting two places P and Q, but it can only hold two people at a time. They have one torch and, because it's night, the torch has to be used when crossing the bridge. Person A can cross the bridge in 1 minute, B in 2 minutes, C in 5 minutes, and D in 8 minutes. When two people cross the bridge together, they must move at the slower person's pace. Can they all get across the bridge from P to Q if the torch lasts only 15 minutes?

2 Fundamentals 3

2.1 Domino Tilings



Problem 5. The square below has two holes in it. Answer Y if the corresponding square can be made with dominoes. Answer N otherwise. Explain why.

2.2 Chicken McNugget Theorem (or Frobenius Coin Problem)

Problem 6. What is the largest number of grams that cannot be balanced with only 3-gram and 5-gram weights?

3 Fundamentals 4

3.1 Number Bases

Problem 7. Convert the base-2 number 1010011001 into base-10.

Problem 8. Convert the base-10 number 500 into base-2.

3.2 Probabilities and Expected Values

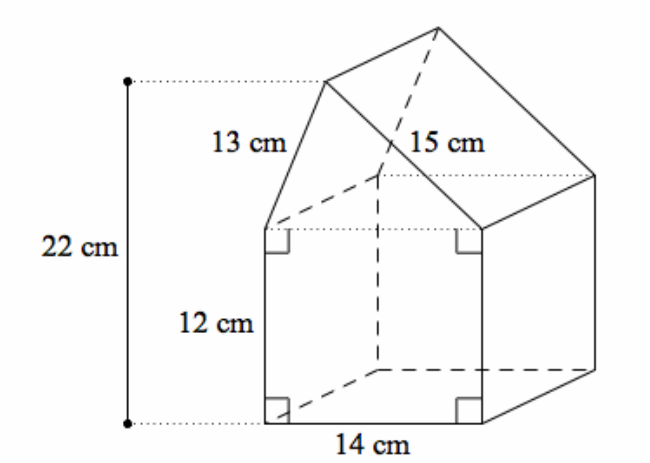
Problem 9. William rolls a pair of dice 1,800 times. About how many of those times do you expect William to roll a sum that is a prime number? Circle the correct range below.

0-100 100-201 201-300 401-600 601-900 901-1300 1301-1800

4 Fundamentals 5

4.1 Surface Area and Volume

Problem 10. The bases of a pentagonal prism are each composed of a triangle and a rectangle as shown below. The volume of the prism is 2380 cubic cm.



(a) What is the height of the prism?

(b) What is the surface area of the prism?

4.2 Linear Equations

Problem 11. Solve the equation $7x - (x - 6) \cdot 5 = 84 - 4x$ for x.

4.3 Statistics

Problem 12. In Computing Wisdom's Fundamentals 5B class, seven students are 9-year-old, 3 students are 11-year-old, and the rest are 12-year-old. If the average student age of the class is 10, how many students in the class are 12-year-old? Solve this problem by (a) writing and solving an equation, and (b) balancing around the average.

4.4 Euclidean Algorithm

Problem 13. Find the GCF of 8010 and 5820 using the Euclidean Algorithm.

4.5 Legendre's Formula

Problem 14. How many trailing zeros are there in 100!?

4.6 Arithmetic Sequences and Series

Problem 15. In an arithmetic sequence, the second term is 7, the 10th term is 61, and the nth term is 655. What is n?

Problem 16. In this problem, we will evaluate the sum of the following arithmetic sequence using two different methods:

$$101, 103, 105, \ldots, 195, 197, 199$$

(a) How many terms are there in the sequence?

(b) Evaluate the sum of the sequence using Gauss's method.

(c) It is known that the nth perfect square is the sum of the first n positive odd integers. That is, $1^2 = 1$, $2^2 = 1 + 3$, $3^2 = 1 + 3 + 5$, and so on. Using this fact, evaluate the sums $1 + 3 + 5 + \cdots + 95 + 97 + 99$ and $1 + 3 + 5 + \cdots + 195 + 197 + 199$.

(d) Evaluate the sum of the sequence using the two sums obtained in (c).

4.7 Iterative Methods

Problem 17. In this problem, we will round $\sqrt{54}$ to the nearest tenth using two methods.

(a) Round $\sqrt{54}$ to the nearest tenth by guess and check method.

(b) Round $\sqrt{54}$ to the nearest tenth by the digit-by-digit square root extraction algorithm.

4.8 Pythagorean Theorem

Problem 18. There are two shortest paths from A to B on the surface of the rectangular prism shown in the diagram below. What is the common length of these shortest paths?